

The influence of dynamical friction on the collapse of spherical density perturbation.

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Abstract

We solve numerically the equations of motion for the collapse of a shell of baryonic matter falling into the central regions of a cluster of galaxies. taking into account of the presence of the substructure inducing dynamical friction. The evolution of the expansion parameter $a(t)$ of the perturbation is calculated in spherical systems. The effect of dynamical friction is to reduce the binding radius and the total mass accreted by the central regions. Using a peak density profile given by Bardeen et al. (1986) we show how the binding radius of the perturbation is modified by dynamical friction. We show how dynamical friction modifies the collapse parameter of the perturbation slowing down the collapse.

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1. Introduction

In the most promising cosmological scenarios structure formation in universe is generated through the growth and collapse of primeval density perturbations originated from quantum fluctuations (Guth & Pi 1982; Hawking 1982; Starobinsky 1982; Bardeen et al. 1986) in an inflationary phase of early universe. perturbations is due to gravitational instability. The statistics of density fluctuations originated in the inflationary era are Gaussian and it can be expressed entirely in terms of the power spectrum of density fluctuations:

$$P(k) = \langle |\delta_{\mathbf{k}}|^2 \rangle \quad (1)$$

where

$$\delta_{\mathbf{k}} = \int d^3x \exp(-i\mathbf{k}\mathbf{x}) \delta(\mathbf{x}) \quad (2)$$

$$\delta(\mathbf{x}) = \frac{\rho(\mathbf{x}) - \rho_b}{\rho_b} \quad (3)$$

and ρ_b is the mean background density. In biased structure formation theory it is assumed that cosmic structures of linear scale R form around the peak of density field, $\delta(\mathbf{x})$, smoothed on the same scale. Density perturbations evolve towards non-linear regime because of gravitational instability and it breaks away from general expansion at:

$$t_m = \left[\frac{3\pi}{32G\rho_b} (1 + \bar{\delta}) \right]^{1/2} (1 + z)^{3/2} \quad (4)$$

where z is the redshift, $\bar{\delta}$ is the overdensity within r . When $\bar{\delta} \simeq 1$ the density perturbation begins to recollapse. The collapse time, T_{c0} , depends on the characteristic of initial fluctuation field as average overdensity and on the environment in which the perturbation is embedded. This last features depends on the cosmological scenario. The mean characteristics of the structure of the structures which form around the peak of the density field depend on the spectrum, $P(k)$, which in turn depends on the matter that dominates the universe (CDM, WDM, HDM). In this context the most succesful model is the biased CDM model (Liddle & Lith 1994) based on a scale invariant spectrum of density fluctuations growing under gravitational instability and on

the hypothesis that the universe is dominated by cold dark matter. A very simple model for accretion of matter in cluster of galaxies was first investigated by Gunn & Gott (1972). In their accretion model the density profile is given by:

$$\begin{aligned} \rho_i &= \rho_{ei} + (\rho_{ci} + \rho_+ - \rho_{ei}) \frac{R_i^3}{r_i^3} & r > R_i \\ \rho_{ci} + \rho_+ & & r_i \leq R_i \end{aligned} \quad (5)$$

where ρ_{ci} and ρ_{ei} are the critical and external density, ρ_+ gives the density surplus present in the radius R_i respect to the critical density. The perturbation with $\rho > \rho_{ci}$ reach a maximum radius $r_m = \frac{x}{\delta}$ at a time:

$$T_{c0}/2 = \frac{\pi}{H_i} \frac{(1 + \bar{\delta})}{\bar{\delta}^{3/2}} \quad (6)$$

where δ is the overdensity in the radius r and H_i is the Hubble parameter at time t_i . After $T_{c0}/2$ the matter collapse and its infall is radial. The perturbation needs a time T_{c0} for the total collapse. Gunn & Gott model is a oversimplification of the perturbation evolution. In Gunn & Gott model there are no tidal interactions among the shell of baryonic matter and the external density perturbations and it is supposed that there is no substructure, (collapsed objects of lenght less than that of main perturbation). One of the features of CDM models is an abundant production of substructure. In this scenario structure formation goes from bottom to up through gravitational clustering, merging and violent relaxation of small scale substructure (White & Rees 1978). So the matter inside a given region is clumped in a hierarchy of objects of various dimensions. The time a given lenght goes nonlinear can be obtained from the condition on mass spectrum, $\sigma_0(M)$:

$$\sigma_0(M) = 1 = 1 + z_{nl} \quad (7)$$

where:

$$\sigma_0(M) = \frac{1}{2\pi^2} \int P(k) k^2 W(kR_f) dk \quad (8)$$

beeing R_f the filtering scale and $W(kR_f)$ the window function:

$$W(kR_f) = \frac{3[\sin(kR_f) - kR_f \cos(kR_f)]}{kR_f^3} \quad (9)$$

So a subgalactic scale collapse at a redshift:

$$1 + z_{nl} = \frac{30}{b} \quad (10)$$

(Silk & Stebbins 1993), where b is the biasing parameter. In particular a perturbation of $10^{15}M_{\odot}$ collapses at a redshift $z \simeq 0.02$ while perturbations in the range $10^6M_{\odot} - 10^9M_{\odot}$ collapse almost at the same $z \simeq 18$ because the mass variance in this region varies only of a factor 3 (Rees 1986). A part of this last perturbations has a cross section too little for gravitational merging to be destroyed (Rees 1986). Therefore a mass perturbation of $10^6M_{\odot} - 10^9M_{\odot}$ can survive until the cluster enters nonlinear phase at $z \simeq 0.02$. Because of the presence of substructure in a cluster Gunn & Gott model needs a revision. In fact substructure acts as a source of stochastic fluctuations in gravitational field and induces dynamical friction (Antonuccio & Colafrancesco 1994) and this produces a modification of the motion of shells of baryonic matter in a density perturbation. In particular dynamical friction delays the collapse of low density perturbations ($\bar{\delta} \simeq 0.01$) (Antonuccio & Colafrancesco 1994). The plan of the paper is as follows. In section 2 by numerical integration of equations of motion of a shell of baryonic matter made of galaxies and substructure of $10^6M_{\odot} - 10^9M_{\odot}$ we show how the expansion parameter of the same varies with time. In section 3 we show how dynamical friction affects the binding radius of a cluster.

2. Modification of the expansion parameter of a shell.

The equation of motion of a shell of baryonic matter around a maximum of the density field, neglecting tidal interactions and substructure, can be expressed in the form:

$$\frac{d^2r}{dt^2} = -\frac{GM}{r^2(t)} \quad (11)$$

(Peebles 1980, eq. 19.9), where M is the mass enclosed in the proper radius $r(t)$. Using Gunn & Gott notation the proper radius can be written as:

$$r(r_i, t) = a(r_i, t)r_i \quad (12)$$

where r_i is the initial radius, $a(r_i, t)$ is the expansion parameter of the shell. At the initial time t_i the initial condition is given by $a(r_i, t_i) = 1$. In presence of substructure it is necessary to modify the equation of motion Eq. (11) because of the graininess of mass distribution in the system, due to substructure. In a material system gravitational field can be decomposed into an average field, $\mathbf{F}_0(r)$, generated from the smoothed out distribution of mass, and a stochastic component, $\mathbf{F}_{stoch}(r)$, generated from the fluctuations in number of the neighbouring particles. The stochastic component of the gravitational field is specified assigning a probability density, $W(\mathbf{F})$, (Chandrasekhar & von Neumann 1942). In a infinite Homogeneous unclustered system $W(\mathbf{F})$ is given by Holtsmark distribution (Chandrasekhar & von Neumann 1943) while in inhomogeneous and clustered systems $W(\mathbf{F})$ is given by Kandrup (1980) and Antonuccio & Barandela (1992) respectively. The stochastic force, \mathbf{F}_{stoch} , in a self-gravitating system modifies the motion of particles as it is done by a frictional force. In fact a particle moving faster than its neighbours produces a deflection of their orbits in such a way that average density is greater in the direction opposite to that of motion causing a slowing down in its motion. The modified equation of motion of a shell of baryonic matter can be written in the form:

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{GM}{r^2(t)} - \eta\mathbf{v} \quad (13)$$

(Langevin 1902, Saslaw 1985) where η is the coefficient of dynamical friction. Supposing that there are no correlations among random force and their derivatives we have:

$$\eta = \frac{\int d^3F W(F) F^2 T(F)}{2 \langle v^2 \rangle} \quad (14)$$

(Kandrup 1980), where $T(F)$ is the average duration of a random force impulse, $W(F)$, is the probability distribution of stochastic force that for a clustered system is given by Antonuccio & Barandela (1992). Equation (13) in terms of $a(r_i, t)$ and

$$\rho(r_i, t) = \frac{3M}{4\pi a^3(r_i, t) r_i^3} = \frac{\rho(r_i, t_i)}{a^3(r_i, t_i)} = \rho_{ci}(1 + \overline{\delta_i}) \quad (15)$$

can be written as:

$$\frac{d^2a}{dt^2} = -\frac{4\pi G\rho_{ci}(1 + \bar{\delta}_i)}{a^2(t)} - \eta\frac{da}{dt} \quad (16)$$

where ρ_{ci} is the background density a time t_i and $\bar{\delta}_i$ is the overdensity within r_i . Using the parameter $\tau = \frac{t}{T_{c0}}$ the Eq. (16) may be written in the form:

$$\frac{d^2a}{d\tau^2} = -\frac{4\pi G\rho_{ci}(1 + \bar{\delta}_i)}{a^2(\tau)}T_{co}^2 - \eta T_{co}\frac{da}{d\tau} \quad (17)$$

Referring to the calculation of η , given by Antonuccio & Colafrancesco (1994) and using equation (17) the time evolution of the expansion parameter, $a(\tau)$, of the shell can be obtained. Equation (17) can be solved looking for an asymptotic expansion. This was made by Antonuccio & Colafrancesco (1994). In their paper they gave only an expression for the collapse time T_c . Equation (17) can be also solved numerically. To this aim we used a Runge-Kutta integrator of 4th order. We studied the motion of a shell of low density, $\bar{\delta} = 0.01$, typical of a perturbation present in the outskirts of a cluster of galaxies. The initial conditions were chosen remembering that in expansion phase the shell moves with Hubble flow and that at the maximum of expansion the initial velocity is zero. In figure 1, we show the expansion parameter $a(\tau)$ versus τ both when the dynamical friction is taken into account and when it is absent. As we can see dynamical friction slows down the collapse of the shell of matter in agreement with the analytic calculation of the collapse time, T_c , of a shell in which dynamical friction effect is taken into account given by Antonuccio & Colafrancesco (1994).

3. Binding radius of a cluster in presence of dynamical friction.

In biased galaxy formation theory structures form around the local maxima of the density field. Every density peak binds a mass m that can be calculated when we know the binding radius of the density peak. The radius of the bound region for a chosen density profile $\bar{\delta}(r)$ may be obtained in several ways. A first criterion is statistic. The binding radius of the region, r_b , is given by the solution of the equation:

$$\langle \bar{\delta}(r) \rangle = \langle (\bar{\delta} - \langle \bar{\delta} \rangle)^2 \rangle^{1/2} \quad (18)$$

(Ryden 1988). At radius $r \ll r_b$ the motion of particles is predominant toward the peak while when $r \gg r_b$ the particle is not bound to the peak. Another criterion that can be used is dynamical. It supposes that the binding radius is given by the condition that a shell collapse in a time, T_c , smaller than the age of the universe t_0 :

$$T_c(r) \leq t_0 \quad (19)$$

(Hoffmann & Shaham 1985). This last criterion, differently from the previous one, contains some prescriptions particularly connected with the physics of the collapse process of a shell. For this reason we used it to calculate the binding radius. The time of collapse, $T_c(r)$, at radius r can be obtained solving numerically equation (16) for different values of $\bar{\delta}_i$ from a given density profile $\bar{\delta}(r)$. We use the average density profile given by Bardeen et al. (1986):

$$\delta(r) = A \left\{ \frac{\nu \xi(r)}{\xi(0)^{1/2}} - \frac{\theta(\nu\gamma, \gamma)}{\gamma \xi(0)^{1/2} (1 - \gamma^2)} \left[\gamma^2 \xi(r) + \frac{R_*^2 \nabla^2 \xi(r)}{3} \right] \right\} \quad (20)$$

where A is a constant given by the normalization of the perturbation spectrum, $P(k)$, $\nu = \frac{\delta(0)}{\sigma_0(M)}$, $\xi(r)$ is the correlation function of two points, γ and R_* two constants obtainable from the spectrum (see Bardeen et al. 1986) and finally $\theta(\gamma\nu, \gamma)$ is a function given in the quoted paper (eq. 6.14). Given the average density profile the average density inside the radius r in a spherical perturbation is given by:

$$\bar{\delta} = \frac{3}{r^3} \int_0^r dx \delta(x) x^2 \quad (21)$$

We calculated the time of collapse, $T_{c0}(r)$, using Eq. (6) with the density profile given in Eq. (20). We repeated the calculation of $T_{c0}(r)$ for $1.5 < \nu < 3$ and we applied the condition given in Eq. (19) to the curves $T_{c0}(r)$ previously obtained. The result is the plot in Fig. 2 for the binding radius r_b versus ν .

4. Conclusion

In the first part of this paper we show how the dynamical friction slow down the collapse of a spherical density perturbation. The effect grows with increasing of the coefficient of dynamical friction η . In the second part, we show how dynamical friction reduces the binding radius (see fig.2).

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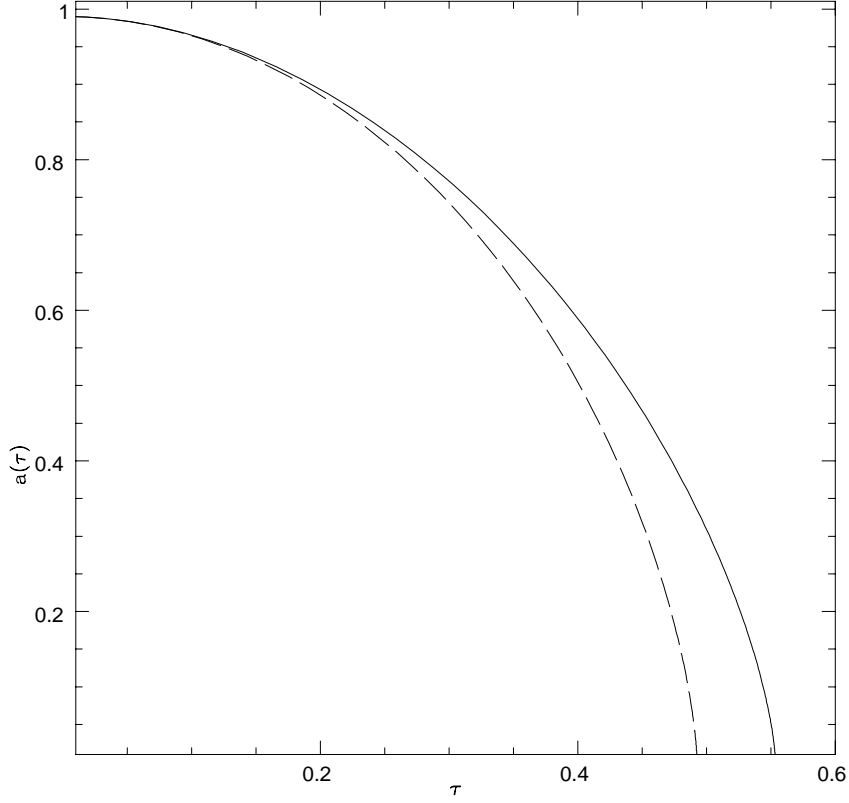


Figure 1: Temporal evolution of the expansion parameter $a(\tau)$ of a shell of matter made of galaxies and substructure versus τ . The dashed line shows $a(\tau)$ when dynamical friction is absent, while the solid line is the same when dynamical friction is taken into account. We assume a cluster radius of $R_{cl} = 5h^{-1}Mpc$, a central overdensity $\bar{\delta} = 0.01$ and a total number of peaks of substructure $N_{tot} = 10^3$.

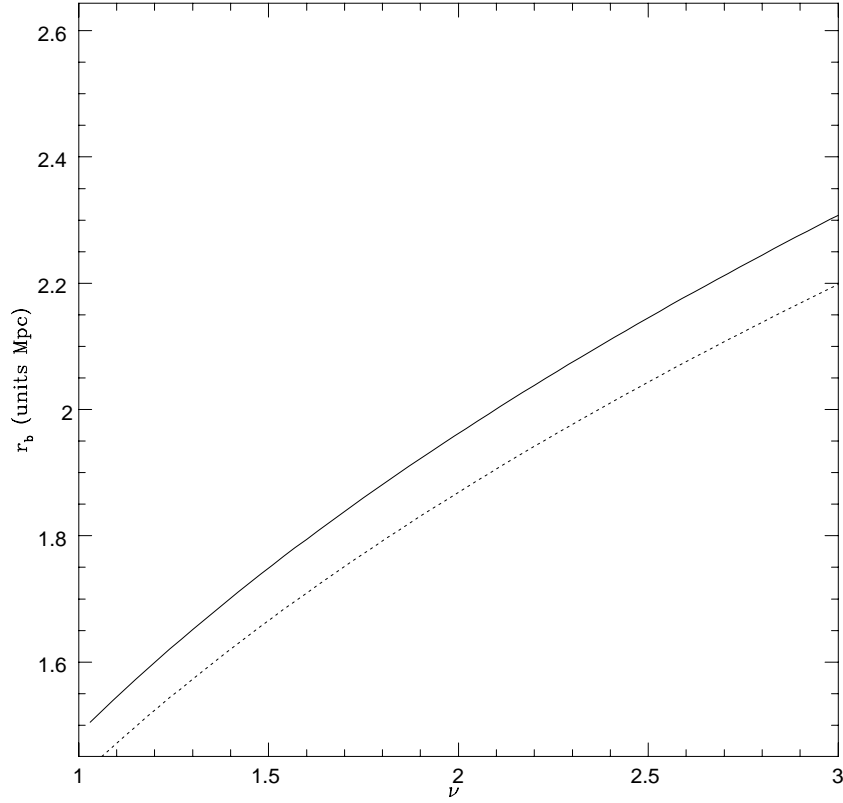


Figure 2: Variation of the binding radius r_b with ν . The solid line is the binding radius in absence of dynamical friction, while the dashed line is the same as in presence of dynamical friction. The filtering radius used is , $R_f = 1 \text{ Mpc}$